Population Variance Harmony Search Algorithm to Solve Optimal Power Flow with Non-Smooth Cost Function

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Abstract. This chapter presents a novel Harmony Search (HS) algorithm used to solve security constrained optimal power flow (OPF) with various generator fuel cost characteristics. HS is a recently developed derivative-free, meta-heuristic optimization algorithm, which draws inspiration from the musical process of searching for a perfect state of harmony. This chapter analyses the evolution of the population-variance over successive generations in HS and thereby draws some important attention regarding the explorative power of HS. This novel methodology of modified population variance harmony search algorithm (PVHS) easily takes care of solving optimal power flow problem even with non-smooth and piecewise cost functions. This PVHS algorithm was tested on the IEEE30 bus system with three different types of cost characteristics and compared with other reported results.

1 Introduction

Optimal power flow is the main tool used for planning an economic operation of power system [1]. In the recent attention in OPF shows the importance of the electric utilities to find the optimal secure operating point corresponding to the each loading condition. The problem of solving OPF involves estimating the optimal solution of control variables like generator real power, generator voltage magnitude and transformer tap settings corresponding to the best objective function. The

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dependent variable includes load bus voltage magnitude, generator reactive power generation, transmission line thermal loading. In general OPF is large scale highly non linear and constrained problem of minimizing the fuel cost.

OPF problem has been solved using many traditional techniques such as non linear programming, quadratic programming, mixed integer programming and interior point method. The literature review on these methods is given in Momoh et al. [2, 3]. The disadvantage of these traditional methods is it cannot be applicable in case of the prohibited operating regions and multiple fuels. It also has higher sensitivity to initial solution, so it may trap into local optima. The difficulties in implementing OPF can be overcome by modern stochastic algorithms such as evolutionary programming (EP) [4], tabu search (TS) [5], improved evolutionary programming (IEP) [6], modified differential evolution (MDE) [7], particle swarm optimization (PSO) [9], genetic algorithm (GA) [10] and simulated annealing (SA) [11].

In 2001, Geem *et al.* proposed Harmony Search (HS) [13], a derivative-free, meta-heuristic algorithm, mimicking the improvisation process of music players. Since its inception, HS has found several applications in a wide variety of practical optimization problems like pipe-network design [14], structural optimization [15], vehicle routing problem [16], water distribution networks [17, 23], combined heat and power economic dispatch problem [18], Dam Scheduling [19] and numerical optimization [20]. The applicability of harmony search algorithm for discrete variable problem is given in [22]. The hybrid version of harmony search with particle swarm optimization applied to water network design is proposed in [24]. In the PVHS [21], the control parameter known as *distance bandwidth (bw)* has been made equal to the standard deviation of the current population. In this chapter we have used this PVHS algorithm to solve optimal power flow problem having various cost characteristics. The algorithm is applied to IEEE 30 bus test system effectively to show the appropriateness of the method. The simulation results with three different cost characteristics are comparable with the recently reported results.

2   OPF Problem Formulation

The objective of OPF problem is to minimize the total fuel cost of generators while satisfying several power system steady state security constraints. If x is the vector of state variables consisting of slack bus real power P_{g1}, load bus voltages V_{Lk}, generator reactive power outputs Q_{gj}, and transmission line thermal loading S_l, x can be expressed as:

$$x^T = [P_{g1}, V_{l1}, \ldots, V_{lNL}, Q_{g1}, \ldots, Q_{gNG}, S_{l1}, \ldots, S_{lNB}] \quad (1)$$

where NL, NG and NB are the number of load buses, the number of generators and the number of transmission lines, respectively. *u* is the vector of control variables consisting of real power outputs P_g except at the slack bus, generator voltages V_g, transformer tap settings T. Hence, *u* can be expressed as:

$$u^T = [P_{g2},....,P_{gNG},V_{g1},....,V_{gNG},T_1,...,T_{NT}] \tag{2}$$

where NT is the number of regulating transformers.

The objective of OPF problem can be expressed as

$$\text{Minimize} \quad F = \sum_{j=1}^{NG} F_i\left(P_{gi}\right) \tag{3}$$

where F is the total generator fuel cost and F_i is the fuel cost of generator connected to ith bus. The system equality constraints $g(x,u)$ is described by the following power balance equation

$$P_{gi} - P_{di} = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\cos\left(\theta_{ij} - \delta_i + \delta_j\right) \quad i = 1,...,N \tag{4}$$

$$Q_{gi} - Q_{di} = -\sum_{j=1}^{N} |V_i||V_j||Y_{ij}|\sin\left(\theta_{ij} - \delta_i + \delta_j\right) \quad i = 1,...,N \tag{5}$$

where P_{gi} is the total real power generation at ith bus, P_{di} is the total real power demand at i^{th} bus, Q_{gi} is the total reactive power generation at i^{th} bus, Q_{di} is the total reactive power demand at i^{th} bus, $|V_i|$ is the voltage magnitude at i^{th} bus, $|V_j|$ is the voltage magnitude at j^{th} bus, $|Y_{ij}|$ is the magnitude of the ij^{th} element of $Ybus$, θ_{ij} angle of the ij^{th} element of $Ybus$, δ_i voltage angle at i^{th} bus and δ_j is the voltage angle at j^{th} bus.

The system inequality constraints $h(x, u)$ is consist of the following

1. *Generator constraints*: The generator real power outputs, reactive power outputs and voltages are bounded to its lower and upper limit.

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \quad i = 1,...,NG \tag{6}$$

$$Q_{gi}^{min} \leq Q_{gi} \leq Q_{gi}^{max} \quad i = 1,...,NG \tag{7}$$

$$\left|V_{gi}^{min}\right| \leq \left|V_{gi}\right| \leq \left|V_{gi}^{max}\right| \quad i = 1,...,NG \tag{8}$$

where P_{gi}^{min} and P_{gi}^{max} are the minimum and maximum real power generation at i^{th} generator bus, Q_{gi}^{min} and Q_{gi}^{max} are the minimum and maximum reactive power generation at i^{th} generator bus, $\left|V_{gi}^{min}\right|$ and $\left|V_{gi}^{max}\right|$ are the minimum and maximum voltage magnitude at i^{th} generator bus.

2. *Transformer constraints*: Transformer tap settings are bounded by minimum and maximum limits as follows

$$T_i^{min} \leq T_i \leq T_i^{max} \quad i = 1,...,NT \tag{9}$$

where $T_i^{\min}$ and $T_i^{\max}$ are the minimum and maximum tap setting limit of ith transformer.

3. *Security constraints*: It includes the limits in the voltage magnitude of load buses and thermal loading limits of all transmission lines as follows.

$$\left|V_{Li}^{\min}\right| \leq \left|V_{Li}\right| \leq \left|V_{Li}^{\max}\right| \quad i = 1,...,NL \tag{10}$$

$$\left|S_{li}\right| \leq S_{li}^{\max} \quad i = 1,...,NB \tag{11}$$

where $\left|V_{Li}^{\min}\right|$ and $\left|V_{Li}^{\max}\right|$ are the minimum and maximum voltage magnitude at i^{th} load bus, $S_{li}^{\max}$ is the thermal limit of ith transmission line.

Constraints Handling: The problem of handling these constraints in the state variables are accommodated in the algorithm by including the constraints violation as quadratic penalty terms in the objective function itself.

$$
\begin{aligned}
F_{corr} = F &+ K_p \left(P_{g1} - P_{g1}^{\lim}\right)^2 + K_v \left(V_{Li} - V_{Li}^{\lim}\right)^2 + \\
&+ K_Q \left(Q_{gi} - Q_{gi}^{\lim}\right)^2 + K_S \left(\left|S_{li}\right| - S_{li}^{\lim}\right)^2
\end{aligned}
\tag{12}
$$

where K_p, K_v, K_Q and K_S are the penalty factors corresponding to slack bus real power generation, load bus voltage magnitude, generator reactive power and transmission line thermal loadings, respectively. In the equation (12) the $x^{\lim}$ is equals to $x_{\min}$ if x is lesser than the minimum limit and $x_{\max}$ if x is greater than maximum limit.

3 Harmony Search algorithm

3.1 Classical Harmony Search Algorithm

In the harmony search algorithm musician improvises the pitches of his/her instrument to obtain a better state of harmony. The different steps of the classical HS algorithm are described below:

Step 1: The 1st step is to specify the problem and initialize the parameter values. The optimization problem is defined as minimize (or maximize) $f(\mathbf{x})$ such that $x_{\min}^i \leq x_i \leq x_{\max}^i$, where $f(\mathbf{x})$ is the objective function, $\mathbf{x}$ is a solution vector consisting of N decision variables (x_i) and $x_{\min}^i$ and $x_{\max}^i$ are the lower and upper bounds of each decision variable, respectively. Other algorithm parameters, such as harmony memory size (*HMS*), or the number of solution vectors in the harmony memory; harmony memory considering rate (*HMCR*); pitch adjusting rate (*PAR*); and the number of improvisations (*NI*) or stopping criterion are also specified in this step.

Step 2: The 2nd step is to initialize the Harmony Memory. The initial harmony memory is generated from a uniform distribution in the ranges [x_{min}^i, x_{max}^i], as

$$x_i^j = x_{min}^i + r \times \left(x_{max}^i - x_{min}^i \right) \tag{13}$$

where $i = 1, 2, .., N$, $j = 1, 2, 3, ..., HMS$, and $r \sim U(0,1)$.

Step 3: The third step is known as the 'improvisation' step. The New Harmony vector $y = (y_1, y_2, ..., y_N)$ is generated by using memory consideration, pitch adjustment, and random selection. The procedure works as follows:

Pseudo-code of improvisation in HS
  for each $i \in [1, N]$ **do**
    if $U(0,1) \leq HMCR$ **then** /*memory consideration*/
      $y_i = x_i^j$, where $j \sim U(1,2,...,HMS)$.
      if $U(0,1) \leq PAR$ **then** /* Pitch adjustment */
$$Y_i = Y_i + r \times bw \text{, where } r \sim U(0,1) \tag{14}$$
    else /* random selection */
$$y_i = x_{min}^i + r \cdot \left(x_{max}^i - x_{min}^i \right) \tag{15}$$
    endif
  done

Step 4: In this step the harmony memory is updated. The generated harmony vector $y = (y_1, y_2, ..., y_N)$ replaces the worst harmony in the HM (harmony memory) only if its fitness is better than the worst harmony.

Step 5: The stopping criterion (generally the number of iterations) is checked. If it is satisfied, computation is terminated. Otherwise, Steps 3 and 4 are repeated.

3.2 Modified Population Variance Harmony Search (PVHS) Algorithm

In [21] Mukhopadhyay et al. analyze the explorative power in HS as follows:

Theorem 1. Let $x = \{x_1, x_2, ..., x_N\}$ be the current population, $Y = \{Y_1, Y_2, ..., Y_N\}$ the intermediate population obtained after harmony memory consideration and pitch adjustment. If HMCR be the harmony memory consideration probability, PAR the pitch-adjustment probability, bw the arbitrary distance bandwidth and if we consider the allowable range for the decision variables (x_i) to be $\{x_{min}, x_{max}\}$ where $x_{max} = a, x_{min} = -a$, then

$$\begin{aligned}
\mathrm{E}\big(\mathrm{var}(Y)\big) = \frac{(m-1)}{m} \cdot [& HMCR \cdot \mathrm{var}(x) + HMCR \cdot (1-HMCR) \cdot \overline{x}^2 \\
& + HMCR \cdot (1-HMCR) \cdot PAR \cdot bw \cdot \overline{x} \\
& + HMCR \cdot PAR \cdot bw^2 \cdot \left(\frac{1}{3} - \frac{HMCR \cdot PAR}{4}\right) + \frac{a^2}{3} \cdot (1-HMCR)]
\end{aligned} \tag{16}$$

If *HMCR* is chosen to be very high (i.e. very near to 1) and the distance band-width parameter (*bw*) is chosen to be the standard deviation of the current population, then population variance (without selection) will grow almost exponentially over generations. Now, Neglecting the terms containing $(1-HMCR)$, and choosing $bw = \sigma(x) = \sqrt{\mathrm{var}(x)}$ the expression (16) becomes:

$$\mathrm{E}(\mathrm{var}(Y)) = \frac{(m-1)}{m} \cdot \left[HMCR + HMCR \cdot PAR \cdot \left(\frac{1}{3} - \frac{HMCR \cdot PAR}{4}\right) \right] \cdot \mathrm{var}(x) \tag{17}$$

From equation (17) it is seen that if we do not include selection in the algorithm, then the expected variance of the gth population (X_g) becomes:

$$\mathrm{E}\big(\mathrm{var}(X_g)\big) = \left\{ \frac{(m-1)}{m} \cdot \left[HMCR + HMCR \cdot PAR \cdot \left(\frac{1}{3} - \frac{HMCR \cdot PAR}{4}\right) \right] \right\}^g \cdot \mathrm{var}(X_0) \tag{18}$$

In equation (10) if we choose the values of the parameters *HMCR*, *PAR* in such a way that the term within the second brackets becomes greater than unity, then we can expect an exponential growth of population variance. This growth of expected population variance over generations gives the algorithm a strong explorative power. In modified HS the *bw* is changed dynamically as

$$\sigma(\mathrm{x}) = \sqrt{\mathrm{var}(\mathrm{x})} \tag{19}$$

We also took *HMCR* = 0.98 and *PAR* = 0.67 to equip the algorithm with more explorative power after performing a series of experiments.

4 Implementation for OPF

The optimal power flow problem is implemented using the PVHS algorithm by taking the control variables *u* in the each harmony memory and equation (12) as objective. The algorithm stops when the current generation exceeds the total number of generation. The parameters are selected as: total number of generation = 100, harmony memory size (HMS) = 50, harmony memory considering rate (HMCR) = 0.98, pitch adjusting rate (PAR) = 0.67. The detailed implementation methodology is described as follows:

Step 1: Initialize harmony memory size (HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR)

Step 2: Initialize harmonic memory and evaluate objective function after running load flow

Step 3: Improvisation of harmony memory by pitch adjustment

Step 4: Run load flow and evaluate the objective function

Step 5: Update the harmony memory with this improvised solution if it is better than worst solution in memory

Step 6: If stopping criteria is met then print the OPF result and stop, otherwise go to step3.

5 Results and Discussion

The PVHS algorithm was tested on *IEEE*30 bus system consists of 6 generating units, 41 transmission lines and 4 tap-changing transformers [8]. The lower and upper limits on independent variables are shown in Table 1. In all the cases bus 1 is considered as swing bus. The simulation was done by taking a quadratic cost curve in case 1, a piecewise quadratic cost curve in case 2, and quadratic cost curve with valve point loading in case 3. The result of the PVHS algorithm is compared with NLP [8], EP [4], TS [5], PSO [9], IEP [6] and MDE [7]. The algorithm is coded on Intel Pentium IV 2.3 GHz processor and 2 GB RAM memory using Matlab 7.4 [12] programming language.

Table 1 Cost Results of PVHS Algorithm

| Parameter	Lower Limit	Upper Limit	Case1	Case2	Case3
P_{g1}	50	200	176.1824	139.9997	197.5413
P_{g2}	20	80	48.8268	54.9998	52.0571
P_{g5}	15	50	21.5131	24.0997	15.0000
P_{g8}	10	35	22.126	34.9994	10.0000
P_{g11}	10	30	12.224	18.4566	10.0000
P_{g13}	12	40	12.0011	17.9266	12.0000
V_{g1}	0.95	1.05	1.0500	1.0500	1.0333
V_{g2}	0.95	1.10	1.0381	1.0403	1.0100
V_{g5}	0.95	1.10	1.0114	1.0145	0.9657
V_{g8}	0.95	1.10	1.0192	1.0256	1.0307
V_{g11}	0.95	1.10	1.0866	1.0786	1.0981
V_{g13}	0.95	1.10	1.0847	1.0837	1.1000
T_{11}	0.90	1.10	1.0247	1.0059	1.0978
T_{12}	0.90	1.10	0.9267	0.9482	1.0983
T_{15}	0.90	1.10	0.9993	1.0033	1.0703
T_{36}	0.90	1.10	0.9422	0.9463	1.0141
P_{loss}			9.4734	7.0818	13.1984
Sum(P_g)			292.8734	290.4818	296.5984
Penalty			0.0000	0.0000	0.0004
Best Cost			**802.3764**	**647.8126**	**930.723**

5.1 Case 1

In this case the fuel cost characteristics of all the 6 generating units are given by quadratic cost function as

$$f = \sum_{i=1}^{NG} F_i\left(P_{gi}\right) = \sum_{i=1}^{NG} a_i + b_i P_{gi} + c_i P_{gi}^2 \tag{20}$$

where a_i, b_i and c_i are the cost coefficients of the ith generator. The generator cost coefficients are found in [6] and the optimized parameters corresponding to minimum cost is given in Table 1. The results of the PVHS algorithm is compared in Table 2 with other reported results. The statistical results of 50 trials are also reported in Table 2. The algorithm converges quickly and the results are better than others. The convergence characteristic of PVHS algorithm for this case is shown in Figure 1.

Table 2 Cost Comparison with Other Methods for Case 1

Parameter	NLP[8]	EP[4]	IEP[6]	MDE [7]	PVHS
Best Cost	802.40	802.62	802.465	802.376	802.3764
Worst cost	-	805.61	802.581	802.404	802.3912
Avg cost	-	803.51	802.521	802.382	802.3805
Std cost	-	-	0.039	-	0.0135
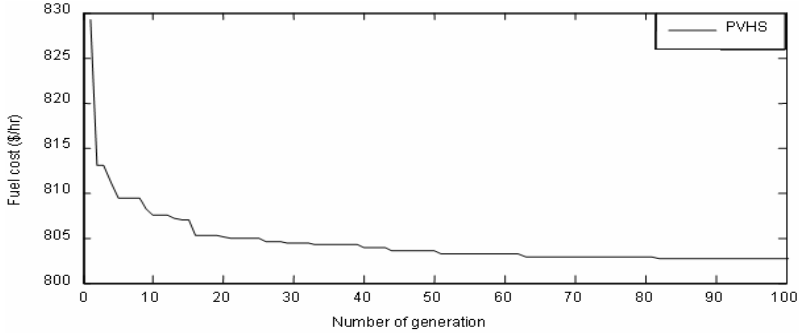
Fig. 1 Convergence of PVHS Algorithm for Case 1

5.2 Case 2

In this case the fuel cost characteristics of the generating units connected at bus 1 and bus 2 are having piecewise quadratic cost curve [6] to model different fuels.

$$F_i\left(P_{gi}\right) = \begin{cases} a_{i1} + b_{i1} P_{gi} + c_{i1} P_{gi}^2, & P_{gi}^{\min} \le P_{gi} \le P_{gi}^1 \\ a_{i2} + b_{i2} P_{gi} + c_{i2} P_{gi}^2, & P_{gi}^1 < P_{gi} \le P_{gi}^2 \\ \quad \cdots\cdots \\ a_{ik} + b_{ik} P_{gi} + c_{ik} P_{gi}^2, & P_{gi}^{k-1} < P_{gi} \le P_{gi}^{\max} \end{cases} \tag{21}$$

where a_{ik}, b_{ik} and c_{ik} are the cost coefficients of the ith generator at the kth interval. The other 4 generators are having same quadratic cost curve coefficients as mentioned in Case 1. The generator cost coefficients are found in [6] and the optimized parameters corresponding to minimum cost is given in Table 1. The results of the PVHS algorithm are compared in Table 3 with other reported results using modified differential evolution algorithm (MDE) [7]. The algorithm converges quickly and the results are better than the other.

Table 3 Cost Comparison with Other Methods for Case2

Parameter	MDE[7]	PVHS
Best Cost	647.846	647.8126
Worst cost	650.664	648.8110
Avg cost	648.356	648.2448
Std cost	-	0.2681

5.3 Case 3

In this case the fuel cost characteristics of the generating units connected at bus 1 and bus 2 are also having a sine component to model the valve point loading effect of the generators as

$$F_i\left(P_{gi}\right) = a_i + b_i P_{gi} + c_i P_{gi}^2 + \left| d_i \sin\left(e_i\left(P_{gi}^{\min} - P_{gi}\right)\right)\right| \qquad (22)$$

where a_i, b_i, c_i, d_i and e_i are the cost coefficients of the ith generating unit. The other 4 generators are having same quadratic cost curve coefficients as mentioned in Case 1. The generator cost coefficients are found in [6] and the optimized parameters corresponding to minimum cost is given in Table 1. The results of the PVHS algorithm are compared in Table 4 with other reported results using improved evolutionary programming (IEP) [6], and modified differential evolution algorithm (MDE) [7]. The algorithm converges quickly and the results are better than the others.

Table 4 Cost Comparison with Other Methods for Case 3

Parameter	IEP[6]	MDE[7]	PVHS
Best Cost	953.573	930.793	930.7237
Worst cost	958.263	954.073	930.7764
Avg cost	956.460	942.501	930.7380
Std cost	1.720	-	0.0162

6 Summary and Conclusions

In this chapter, detailed discussion is carried out about the application of population variance harmony search algorithm to solve the optimal power flow problem

in the presence of security constraints. The algorithm explores the search space quickly with the help of population variance parameter. The PVHS algorithm was tested with IEEE 30 bus test system having three different types of cost characteristics. The comparison of obtained results with other previously reported results shows the effectiveness of the algorithm.

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